Government College of Engineering, Aurangabad

(An autonomous Institute of Government of Maharashtra)

F.E.(All)(Rev) Examination

End semester Examination

GE 141: Engineering Mathematics - I

Time: Three hours 28 NOV 2016 *Max. Marks: 60*

"Verify the course code and check whether you have got the correct question paper".

N.B:-

1. All questions are compulsory
2. Figures to the right indicate full marks
3. Assume suitable data if necessary and state it clearly
4. Use of non programmable calculator is allowed

Q 1: Attempt any three of the following (12)

a) State DeMoivre's theorem and use it to expand $cos7\theta$ in powers of $sin\theta$ & $cos\theta$.

b) Solve the equation $(x+i)^8 + x^8 = 0$

c) If $\log(\log(x+iy)) = p + iq$ then show that $y = x\,tan(tanq\ log\sqrt{x^2+y^2})$

d) If $6\,sinhx + 2coshx + 7 = 0$, find $tanhx$.

Q 2: Attempt any three of the following (12)

a) Examine whether the vectors X_1, X_2, X_3 are linearly dependent or independent. If dependent find the relation between them $X_1 = [3,1,1]$, $X_2 = [2,0,-1], X_3 = [4,2,1]$

b) Test for consistency and solve them if consistent
$5x + 3y + 7z = 4, 2y + 11z + 7x = 5, 3x + 26y + 2z = 9$

c) State Cayley Hamilton theorem and use it to find A^8 where $A = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$

d) Define the rank of the matrix. Find the rank of the matrix by reducing it to its normal form $\begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

Q 3: Attempt any three of the following (12)

a) State Maclaurin's series, Prove that $tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

b) If $y = cos^4 x$, find $\frac{d^{10}y}{dx^{10}}$ at $x = 0$

c) Test the convergence of the infinite series $\sum \frac{n!}{n^n}$

d) Evaluate $\lim_{x\to 0}\left[\frac{a}{x} - cot\left(\frac{x}{a}\right)\right]$

Q 4: Attempt any three of the following (12)

a) find $\frac{dy}{dx}$ if $x^y = y^x$ using partial derivatives

b) if $z = (x^2 - y^2)f(xy)$ then show that $\frac{\partial^2 z}{\partial x \partial y} = (x^2 - y^2)\{3f'(xy) + xyf''(xy)\}$

c) if $u(x^2y^3, z - 3x) = 0$, show that $3x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 9x$

d) if $u = x^2 sin^{-1}\left(\frac{y}{x}\right) - y^2 cos^{-1}\left(\frac{x}{y}\right)$, determine the value of $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$ at the point (1,1).

Q 5: Attempt any three of the following (12)

a) The period T of simple pendulum is $T = 2\pi\sqrt{l/g}$ find the maximum error in T due to possible errors upto 1% in l and 2.5% in g.

b) If $u = x + y - z, uv = y + z, uvw = z$ then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

c) Examine the function $u = x^3 + y^3 - 3xy$ for its extreme values

d) If $u = x + y^2, v = y + z^2, w = z + x^2$ prove that $\frac{\partial x}{\partial u} = -\frac{1}{1+8xyz}$